

STRUCTURES AND TRANSFORMATIONS

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Automorphism of a structure preserves the structure

Conversely, if we start with the automorphism group, we can extract the invariants, which we can identify with the structure.

ex. Klein's Erlangen Programme in Geometry

Geometry is the study of the invariants of some specified transformation group.

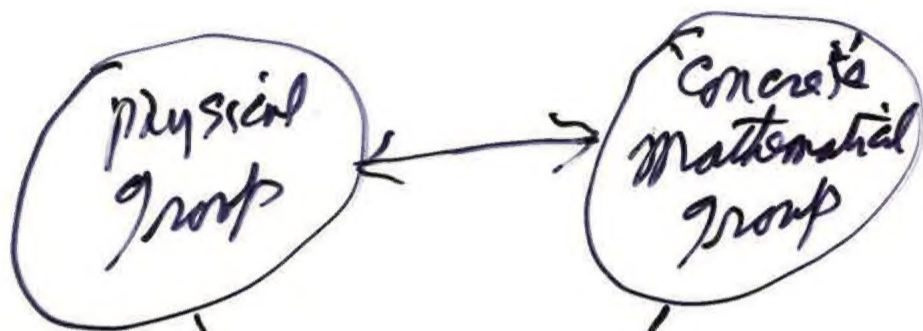
e.g. Euclidean geometry \leftrightarrow Euclidean group
Affine geometry \leftrightarrow Affine "
Projective geometry \leftrightarrow Projective "

More generally, morphisms between structures is the subject matter of Category theory.

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Semantic Approach to Theories in Mathematical Physics

Ex



Both satisfy its formal
axioms of an abstract
group presented syntactically

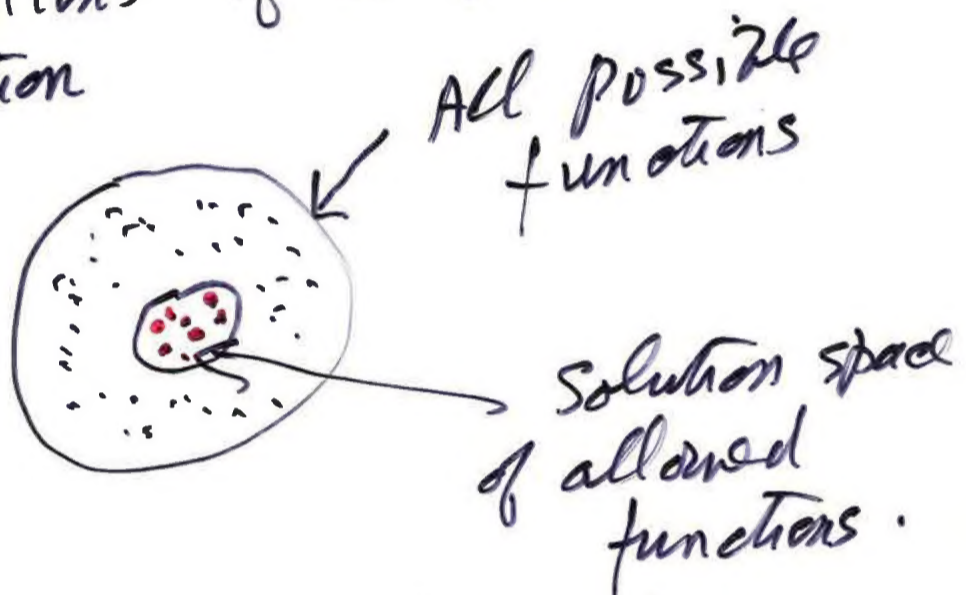
A THEORY OF THEORIES

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Possible states of a physical system associated with points in a function space.

Physical laws impose constraints on possible states (functions)

Ex Solutions of a mathematical equation



Symmetries of the physical theory are automorphisms of the solution space, i.e. map one solution onto another.

Limiting Relations

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ex $Z = X + \alpha Y = 0 \quad (1)$

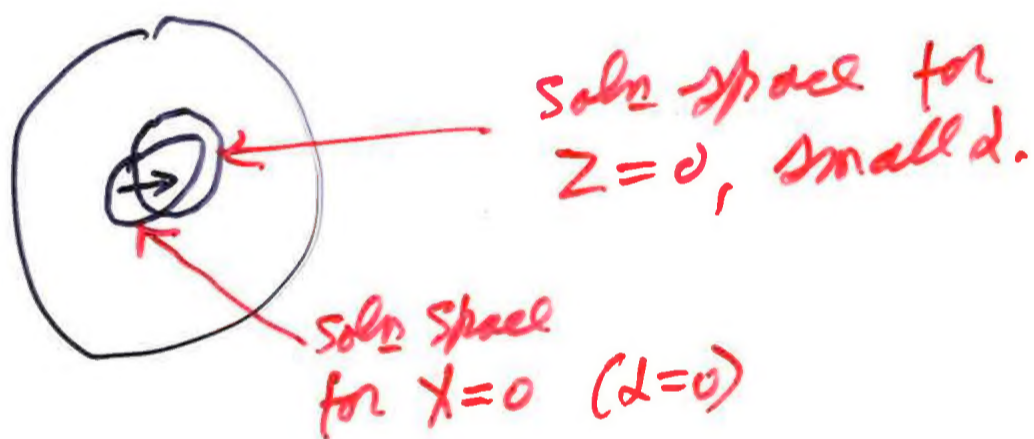
For $\alpha = 0$, eq. becomes.

$$X = 0 \quad (2)$$

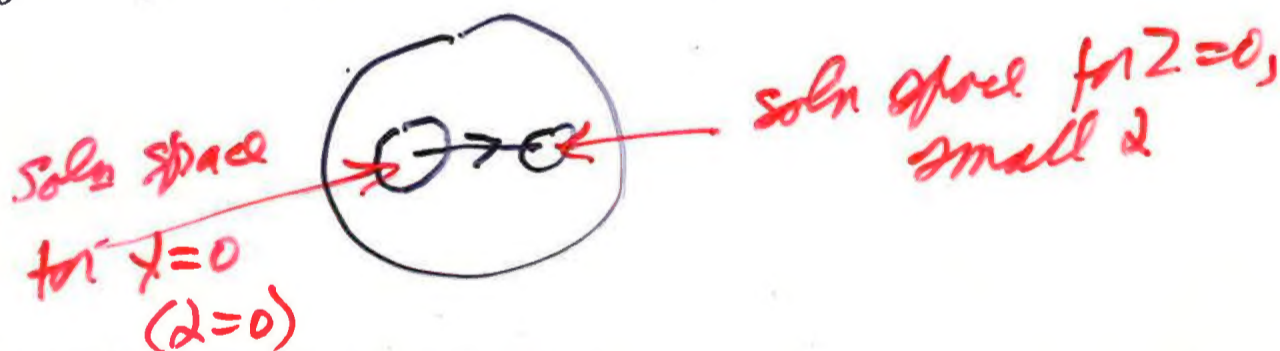
How are solutions of (1) for small α related to solutions of (2) where $\alpha = 0$.

Two possibilities:

(a) Continuous behaviour



(b) Discontinuous behaviour

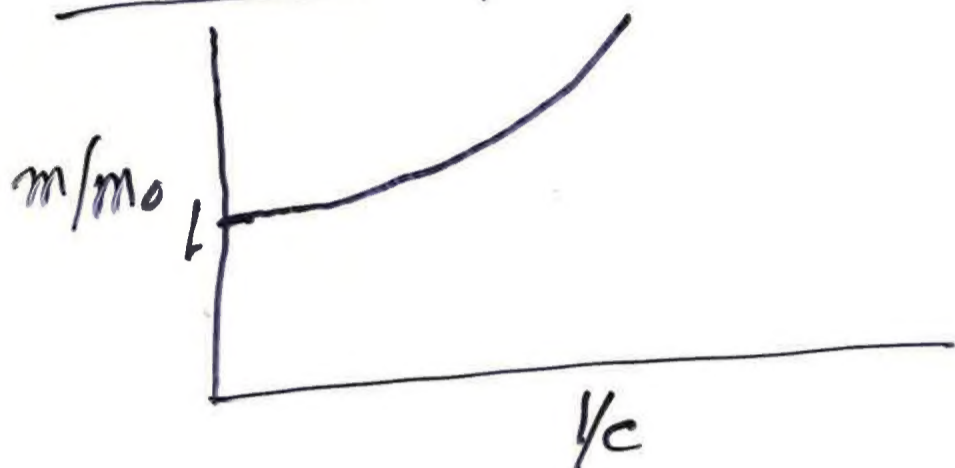


Special Relativity as $1/c \rightarrow 0$

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(a) Consider $m/m_0 = 1/\sqrt{1-v^2/c^2}$

For fixed v , this behaves in a continuous fashion as $1/c \rightarrow 0$



(2) Consider covariant metric tensor

$$g_{\mu\nu} = \begin{pmatrix} 1 & -1/c^2 & & \\ & -1/c^2 & & \\ & & -1/c^2 & \\ & & & -1/c^2 \end{pmatrix}$$

with $1/c = 0$ this becomes

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ & 0 & 0 & 0 \\ & & 0 & 0 \\ & & & 0 \end{pmatrix}$$

which has no inverse, so contravariant metric tensor $g^{\mu\nu}$ does not exist!

g^{ii} exhibits discontinuity at $1/c = 0$





9 marks

Consider pulse of length l , unit amplitude
travelling to the right

For resolution time T , mean
displacement $\bar{\phi}$ satisfies

$$\bar{\phi} < \frac{l}{cT}, \quad \text{for } T > \frac{l}{c}$$

$$< 1$$

Indeed for fixed T , as $\frac{1}{c} \rightarrow 0$

$$\bar{\phi} \rightarrow 0$$

But for any value of $\frac{1}{c}$, however small,
we can always choose T small enough
($< l/c$) so as to make $\bar{\phi} = 1$
for some value of x and t .

Compare fundamental mode with the zero
mode at a particular point.

$$\int_0^T (\sin vt - 0) dt = \frac{1}{v} (1 - \cos vT) \rightarrow 0$$

as $v = \frac{c}{2L} \rightarrow \infty$.

So we have convergence in the mean, but not
pointwise convergence.

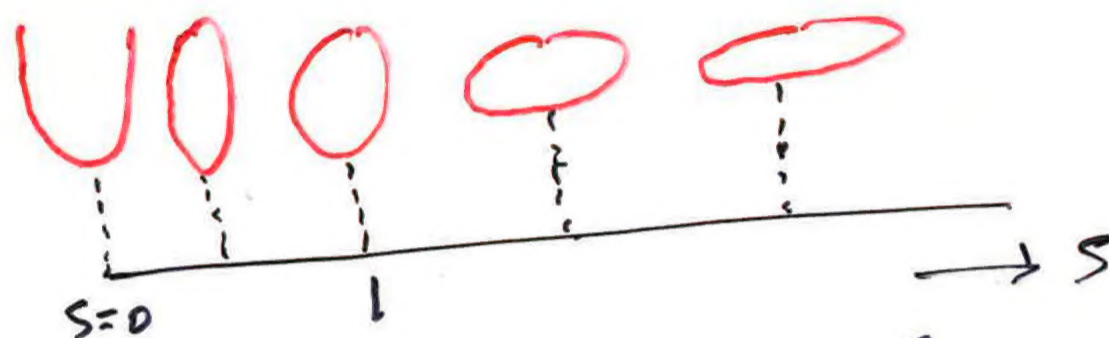
Conic Sections and Structural Stability

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Consider the following eq. in 2-dimensional Euclidean space

$$x^2 + sy^2 - 2x - 2y + 1 = 0$$

for variable parameter $s \geq 0$



For $s=0$ we have a parabola
For $s>0$ ellipses of varying eccentricity
($s=1$ gives a circle).

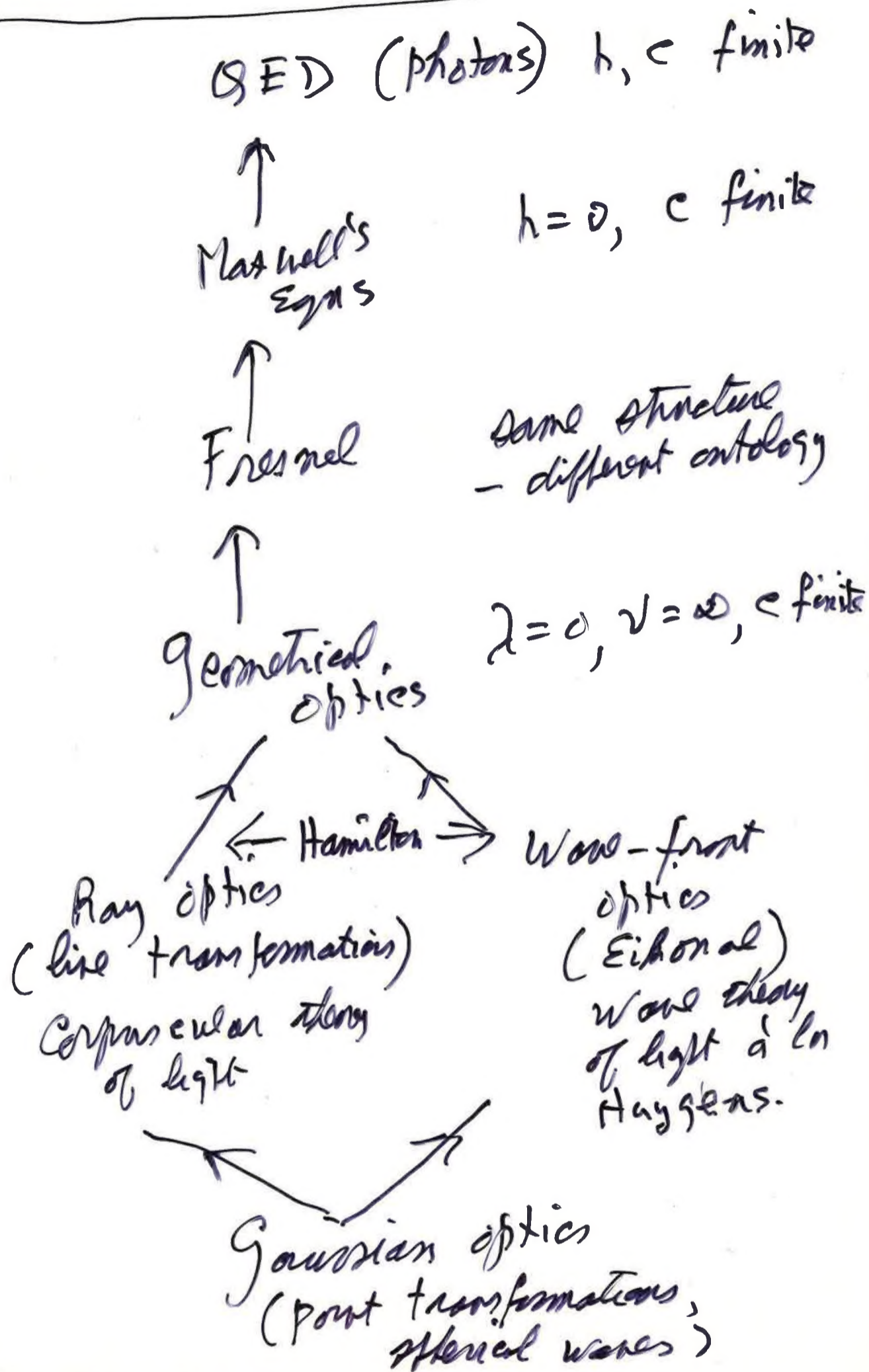
What we are dealing with here is a structural-valued function over s

Relative to the Euclidean group all the figures are in different congruence classes.

But relative to the Affine group all the ellipses are equivalent, but not the parabola, so we have structural stability relative to the affine group for all $s>0$, with a singularity at $s=0$ (catastrophe)

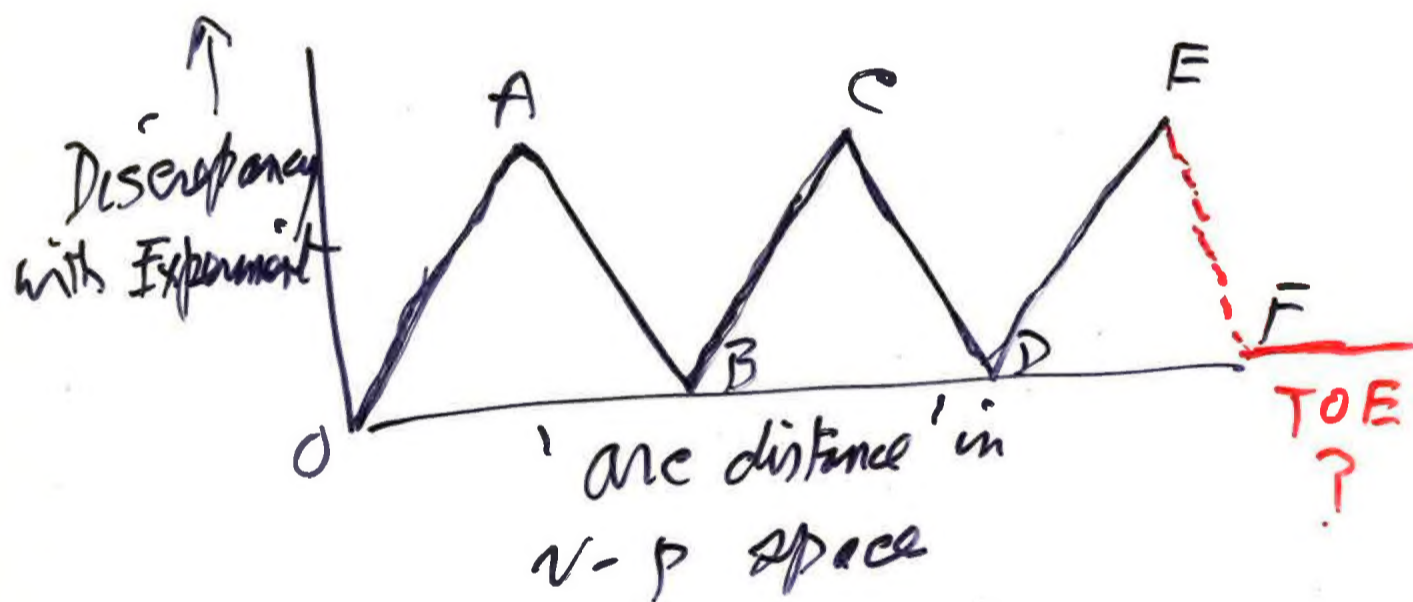
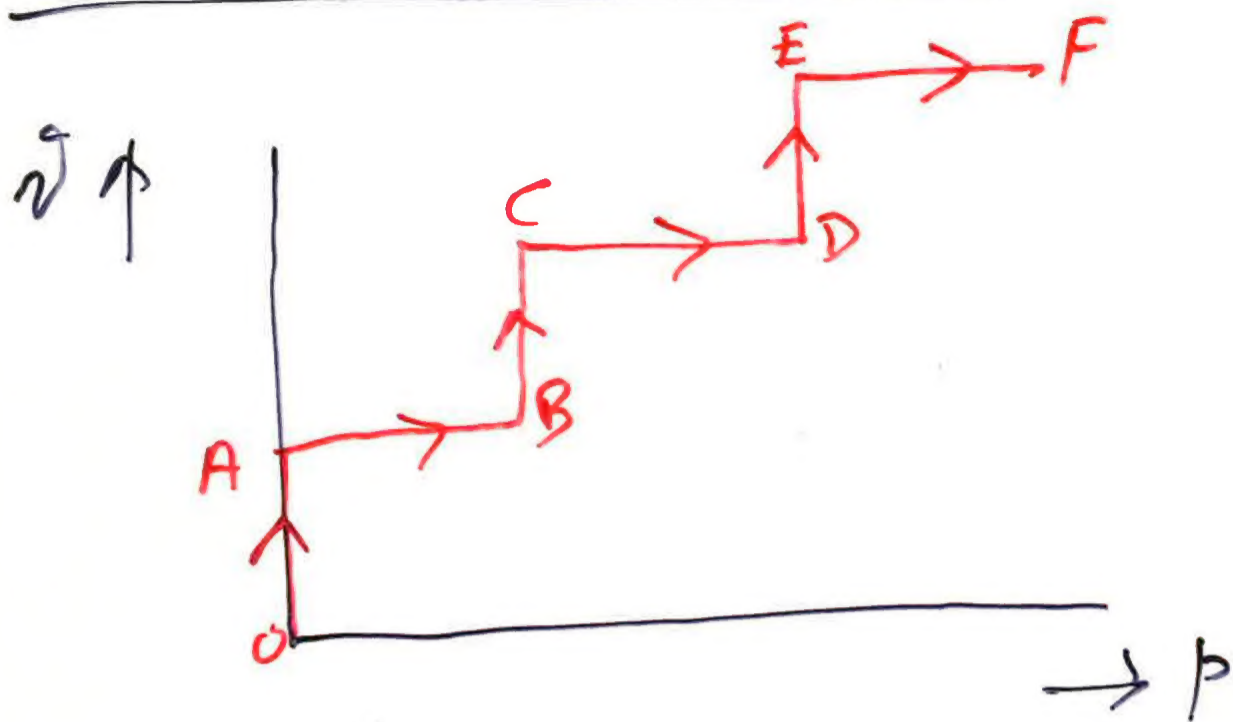
HISTORY OF OPTICS

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PROGRESS IN PHYSICS

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Gauge Transformations and Surplus Structure

$$\rho = e \psi^* \psi \quad \text{and} \quad j = \frac{1}{2} i e \left(\psi^* \frac{d\psi}{dx} - \psi \frac{d\psi^*}{dx} \right)$$

are real quantities and represent physical magnitudes of charge and current density.

ρ and j are invariant under global phase transformations

$$\psi \rightarrow \psi e^{i\alpha}$$

But to retain local gauge invariance under $\psi \rightarrow \psi e^{i\alpha(x)}$ for the current density j , we must replace d/dx by $d/dx - iA(x)$ where A transforms as $A \rightarrow A + \frac{d}{dx}\alpha(x)$ and $j = \frac{1}{2} \cdot i e \left(\psi^* (d/dx - iA) \psi - \psi (d/dx + iA) \psi^* \right)$